Parallel Computing

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The Sage Development Team

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CONTENTS

CHAPTER

ONE

DECORATE INTERFACE FOR PARALLEL COMPUTATION

class sage.parallel.decorate.**Fork**(*timeout=0*, *verbose=False*)

Bases: object

A fork decorator class.

class sage.parallel.decorate.**Parallel**(*p_iter='fork'*, *ncpus=None*, ***kwds*)

Bases: object

Create a parallel-decorated function. This is the object created by *parallel()*.

class sage.parallel.decorate.**ParallelFunction**(*parallel*, *func*)

Bases: object

Class which parallelizes a function or class method. This is typically accessed indirectly through Parallel.__call__().

sage.parallel.decorate.**fork**(*f=None*, *timeout=0*, *verbose=False*)

Decorate a function so that when called it runs in a forked subprocess.

This means that it will not have any in-memory side effects on the parent Sage process. The pexpect interfaces are all reset.

INPUT:

- f – a function
- timeout – (default: 0) if positive, kill the subprocess after this many seconds (wall time)
- verbose – boolean (default: False); whether to print anything about what the decorator does (e.g., killing the subprocess)

Warning

The forked subprocess will not have access to data created in pexpect interfaces. This behavior with respect to pexpect interfaces is very important to keep in mind when setting up certain computations. It's the one big limitation of this decorator.

EXAMPLES:

We create a function and run it with the fork decorator. Note that it does not have a side effect. Despite trying to change the global variable a below in g, the variable a does not get changed:

```
sage: a = 5
sage: @fork
....: def g(n, m):
```

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```
....:     global a
....:     a = 10
....:     return factorial(n).ndigits() + m
sage: g(5, m=5)
8
sage: a
5
```

```
>>> from sage.all import *
>>> a = Integer(5)
>>> @fork
... def g(n, m):
...     global a
...     a = Integer(10)
...     return factorial(n).ndigits() + m
>>> g(Integer(5), m=Integer(5))
8
>>> a
5
```

We use `fork` to make sure that the function terminates after 100 ms, no matter what:

```
sage: @fork(timeout=0.1, verbose=True)
....: def g(n, m): return factorial(n).ndigits() + m
sage: g(10^7, m=5)
Killing subprocess ... with input ((10000000,), {'m': 5}) which took too long
'NO DATA (timed out)'
```

```
>>> from sage.all import *
>>> @fork(timeout=RealNumber('0.1'), verbose=True)
... def g(n, m): return factorial(n).ndigits() + m
>>> g(Integer(10)**Integer(7), m=Integer(5))
Killing subprocess ... with input ((10000000,), {'m': 5}) which took too long
'NO DATA (timed out)'
```

We illustrate that the state of the pexpect interface is not altered by forked functions (they get their own new pexpect interfaces!):

```
sage: # needs sage.libs.pari
sage: gp.eval('a = 5')
'5'
sage: @fork()
....: def g():
....:     gp.eval('a = 10')
....:     return gp.eval('a')
sage: g()
'10'
sage: gp.eval('a')
'5'
```

```
>>> from sage.all import *
>>> # needs sage.libs.pari
```

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```
>>> gp.eval('a = 5')
'5'
>>> @fork()
... def g():
...     gp.eval('a = 10')
...     return gp.eval('a')
>>> g()
'10'
>>> gp.eval('a')
'5'
```

We illustrate that the forked function has its own pexpect interface:

```
sage: gp.eval('a = 15')                                                    #␣
↪needs sage.libs.pari
'15'
sage: @fork()
....: def g(): return gp.eval('a')
sage: g()                                                                  #␣
↪needs sage.libs.pari
'a'
```

```
>>> from sage.all import *
>>> gp.eval('a = 15')                                                      #␣
↪needs sage.libs.pari
'15'
>>> @fork()
... def g(): return gp.eval('a')
>>> g()                                                                    #␣
↪needs sage.libs.pari
'a'
```

We illustrate that segfaulting subprocesses are no trouble at all:

```
sage: cython('def f(): print(<char*>0)')                                   #␣
↪needs sage.misc.cython
sage: @fork
....: def g():
....:     os.environ["CYSIGNALS_CRASH_NDEBUG"]="yes" # skip enhanced backtrace␣
↪(it is slow)
....:     f()
sage: print("this works"); g()                                             #␣
↪needs sage.misc.cython
this works...

------------------------------------------------------------------------
Unhandled SIG...
------------------------------------------------------------------------
'NO DATA'
```

```
>>> from sage.all import *
>>> cython('def f(): print(<char*>0)')                                     #␣
```

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```
↪needs sage.misc.cython
>>> @fork
... def g():
...     os.environ["CYSIGNALS_CRASH_NDEBUG"]="yes" # skip enhanced backtrace (it␣
↪is slow)
...     f()
>>> print("this works"); g()                                                #␣
↪needs sage.misc.cython
this works...
<BLANKLINE>
------------------------------------------------------------------------
Unhandled SIG...
------------------------------------------------------------------------
'NO DATA'
```

sage.parallel.decorate.**normalize_input**(*a*)

Convert `a` to a pair `(args, kwds)` using some rules:

- if already of that form, leave that way.
- if `a` is a tuple make `(a,{})`
- if `a` is a dict make `(tuple(),a)`
- otherwise make `((a,),{})`

INPUT:

- `a` – object

OUTPUT:

- `args` – tuple
- `kwds` – dictionary

EXAMPLES:

```
sage: sage.parallel.decorate.normalize_input( (2, {3:4}) )
((2, {3: 4}), {})
sage: sage.parallel.decorate.normalize_input( (2,3) )
((2, 3), {})
sage: sage.parallel.decorate.normalize_input( {3:4} )
((), {3: 4})
sage: sage.parallel.decorate.normalize_input( 5 )
((5,), {})
```

```
>>> from sage.all import *
>>> sage.parallel.decorate.normalize_input( (Integer(2), {Integer(3):Integer(4)})␣
↪)
((2, {3: 4}), {})
>>> sage.parallel.decorate.normalize_input( (Integer(2),Integer(3)) )
((2, 3), {})
>>> sage.parallel.decorate.normalize_input( {Integer(3):Integer(4)} )
((), {3: 4})
>>> sage.parallel.decorate.normalize_input( Integer(5) )
((5,), {})
```

sage.parallel.decorate.**parallel**(*p_iter='fork'*, *ncpus=None*, ***kwds*)

This is a decorator that gives a function a parallel interface, allowing it to be called with a list of inputs, whose values will be computed in parallel.

Warning

The parallel subprocesses will not have access to data created in pexpect interfaces. This behavior with respect to pexpect interfaces is very important to keep in mind when setting up certain computations. It's the one big limitation of this decorator.

INPUT:

- `p_iter` – parallel iterator function or string: - `'fork'` – (default) use a new forked subprocess for each input - `'multiprocessing'` – use multiprocessing library - `'reference'` – use a fake serial reference implementation
- `ncpus` – integer; maximal number of subprocesses to use at the same time
- `timeout` – number of seconds until each subprocess is killed (only supported by `'fork'`; zero means not at all)
- `reseed_rng`: reseed the rng (random number generator) in each subprocess

Warning

If you use anything but `'fork'` above, then a whole new subprocess is spawned, so none of your local state (variables, certain functions, etc.) is available.

EXAMPLES:

We create a simple decoration for a simple function. The number of cpus (or cores, or hardware threads) is automatically detected:

```
sage: @parallel
....: def f(n): return n*n
sage: f(10)
100
sage: sorted(list(f([1,2,3])))
[(((1,), {}), 1), (((2,), {}), 4), (((3,), {}), 9)]
```

```
>>> from sage.all import *
>>> @parallel
... def f(n): return n*n
>>> f(Integer(10))
100
>>> sorted(list(f([Integer(1),Integer(2),Integer(3)])))
[(((1,), {}), 1), (((2,), {}), 4), (((3,), {}), 9)]
```

We use exactly two cpus:

```
sage: @parallel(2)
....: def f(n): return n*n
```

```
>>> from sage.all import *
>>> @parallel(Integer(2))
... def f(n): return n*n
```

We create a decorator that uses three subprocesses, and times out individual processes after 10 seconds:

```
sage: @parallel(ncpus=3, timeout=10)
....: def fac(n): return factor(2^n-1)
sage: for X, Y in sorted(list(fac([101,119,151,197,209]))): print((X,Y))        #␣
↪needs sage.libs.pari
(((101,), {}), 7432339208719 * 341117531003194129)
(((119,), {}), 127 * 239 * 20231 * 131071 * 62983048367 * 131105292137)
(((151,), {}), 18121 * 55871 * 165799 * 2332951 * 7289088383388253664437433)
(((197,), {}), 7487 * 268288039979128869297108670418919894904868938457124488833)
(((209,), {}), 23 * 89 * 524287 * 94803416684681 * 1512348937147247 *␣
↪53469505413239602323196 57)

sage: @parallel('multiprocessing')
....: def f(N): return N^2
sage: v = list(f([1,2,4])); v.sort(); v
[(((1,), {}), 1), (((2,), {}), 4), (((4,), {}), 16)]
sage: @parallel('reference')
....: def f(N): return N^2
sage: v = list(f([1,2,4])); v.sort(); v
[(((1,), {}), 1), (((2,), {}), 4), (((4,), {}), 16)]
```

```
>>> from sage.all import *
>>> @parallel(ncpus=Integer(3), timeout=Integer(10))
... def fac(n): return factor(Integer(2)**n-Integer(1))
>>> for X, Y in sorted(list(fac([Integer(101),Integer(119),Integer(151),
↪Integer(197),Integer(209)]))): print((X,Y))        # needs sage.libs.pari
(((101,), {}), 7432339208719 * 341117531003194129)
(((119,), {}), 127 * 239 * 20231 * 131071 * 62983048367 * 131105292137)
(((151,), {}), 18121 * 55871 * 165799 * 2332951 * 7289088383388253664437433)
(((197,), {}), 7487 * 268288039979128869297108670418919894904868938457124488833)
(((209,), {}), 23 * 89 * 524287 * 94803416684681 * 1512348937147247 *␣
↪53469505413239602323196 57)

>>> @parallel('multiprocessing')
... def f(N): return N**Integer(2)
>>> v = list(f([Integer(1),Integer(2),Integer(4)])); v.sort(); v
[(((1,), {}), 1), (((2,), {}), 4), (((4,), {}), 16)]
>>> @parallel('reference')
... def f(N): return N**Integer(2)
>>> v = list(f([Integer(1),Integer(2),Integer(4)])); v.sort(); v
[(((1,), {}), 1), (((2,), {}), 4), (((4,), {}), 16)]
```

For functions that take multiple arguments, enclose the arguments in tuples when calling the parallel function:

```
sage: @parallel
....: def f(a, b): return a*b
sage: for X, Y in sorted(list(f([(2,3),(3,5),(5,7)]))): print((X, Y))
(((2, 3), {}), 6)
```

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```
(((3, 5), {}), 15)
(((5, 7), {}), 35)
```

```
>>> from sage.all import *
>>> @parallel
... def f(a, b): return a*b
>>> for X, Y in sorted(list(f([(Integer(2),Integer(3)),(Integer(3),Integer(5)),
↪(Integer(5),Integer(7))]))): print((X, Y))
(((2, 3), {}), 6)
(((3, 5), {}), 15)
(((5, 7), {}), 35)
```

For functions that take a single tuple as an argument, enclose it in an additional tuple at call time, to distinguish it as the first argument, as opposed to a tuple of arguments:

```
sage: @parallel
....: def firstEntry(aTuple): return aTuple[0]
sage: for X, Y in sorted(list(firstEntry([((1,2,3,4),),((5,6,7,8),)]))): print((X,
↪ Y))
((((1, 2, 3, 4),), {}), 1)
((((5, 6, 7, 8),), {}), 5)
```

```
>>> from sage.all import *
>>> @parallel
... def firstEntry(aTuple): return aTuple[Integer(0)]
>>> for X, Y in sorted(list(firstEntry([((Integer(1),Integer(2),Integer(3),
↪Integer(4)),),((Integer(5),Integer(6),Integer(7),Integer(8)),)]))): print((X,␣
↪Y))
((((1, 2, 3, 4),), {}), 1)
((((5, 6, 7, 8),), {}), 5)
```

The parallel decorator also works with methods, classmethods, and staticmethods. Be sure to apply the parallel decorator after ("above") either the `classmethod` or `staticmethod` decorators:

```
sage: class Foo():
....:     @parallel(2)
....:     def square(self, n):
....:         return n*n
....:     @parallel(2)
....:     @classmethod
....:     def square_classmethod(cls, n):
....:         return n*n
sage: a = Foo()
sage: a.square(3)
9
sage: sorted(a.square([2,3]))
[(((2,), {}), 4), (((3,), {}), 9)]
sage: Foo.square_classmethod(3)
9
sage: sorted(Foo.square_classmethod([2,3]))
[(((2,), {}), 4), (((3,), {}), 9)]
sage: Foo.square_classmethod(3)
```

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```
9
```

```
>>> from sage.all import *
>>> class Foo():
...     @parallel(Integer(2))
...     def square(self, n):
...         return n*n
...     @parallel(Integer(2))
...     @classmethod
...     def square_classmethod(cls, n):
...         return n*n
>>> a = Foo()
>>> a.square(Integer(3))
9
>>> sorted(a.square([Integer(2),Integer(3)]))
[(((2,), {}), 4), (((3,), {}), 9)]
>>> Foo.square_classmethod(Integer(3))
9
>>> sorted(Foo.square_classmethod([Integer(2),Integer(3)]))
[(((2,), {}), 4), (((3,), {}), 9)]
>>> Foo.square_classmethod(Integer(3))
9
```

By default, all subprocesses use the same random seed and therefore the same deterministic randomness. For functions that should be randomized, we can reseed the random seed in each subprocess:

```
sage: @parallel(reseed_rng=True)
....: def unif(n): return ZZ.random_element(x=0, y=n)
sage: set_random_seed(42)
sage: sorted(unif([1000]*3)) # random
[(((1000,), {}), 444), (((1000,), {}), 597), (((1000,), {}), 640)]
```

```
>>> from sage.all import *
>>> @parallel(reseed_rng=True)
... def unif(n): return ZZ.random_element(x=Integer(0), y=n)
>>> set_random_seed(Integer(42))
>>> sorted(unif([Integer(1000)]*Integer(3))) # random
[(((1000,), {}), 444), (((1000,), {}), 597), (((1000,), {}), 640)]
```

Warning

Currently, parallel methods do not work with the multiprocessing implementation.

CHAPTER

TWO

REFERENCE PARALLEL PRIMITIVES

These are reference implementations of basic parallel primitives. These are not actually parallel, but work the same way. They are good for testing.

sage.parallel.reference.**parallel_iter**(*f*, *inputs*)

Reference parallel iterator implementation.

INPUT:

- `f` – a Python function that can be pickled using the `pickle_function` command
- `inputs` – list of pickleable pairs (args, kwds), where args is a tuple and kwds is a dictionary

OUTPUT: iterator over 2-tuples `(inputs[i], f(inputs[i]))`, where the order may be completely random

EXAMPLES:

```
sage: def f(N, M=10): return N*M
sage: inputs = [((2,3),{}),  (tuple(), {'M':5,'N':3}), ((2,),{})]
sage: set_random_seed(0)
sage: for a, val in sage.parallel.reference.parallel_iter(f, inputs):
....:     print((a, val))
(((2,), {}), 20)
((), {'M': 5, 'N': 3}), 15)
(((2, 3), {}), 6)
sage: for a, val in sage.parallel.reference.parallel_iter(f, inputs):
....:     print((a, val))
((), {'M': 5, 'N': 3}), 15)
(((2,), {}), 20)
(((2, 3), {}), 6)
```

```
>>> from sage.all import *
>>> def f(N, M=Integer(10)): return N*M
>>> inputs = [((Integer(2),Integer(3)),{}),  (tuple(), {'M':Integer(5),'N
↪':Integer(3)}), ((Integer(2),),{})]
>>> set_random_seed(Integer(0))
>>> for a, val in sage.parallel.reference.parallel_iter(f, inputs):
...     print((a, val))
(((2,), {}), 20)
((), {'M': 5, 'N': 3}), 15)
(((2, 3), {}), 6)
>>> for a, val in sage.parallel.reference.parallel_iter(f, inputs):
...     print((a, val))
((), {'M': 5, 'N': 3}), 15)
```

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```
(((2,), {}), 20)
(((2, 3), {}), 6)
```

CHAPTER

THREE

PARALLEL ITERATOR BUILT USING THE `FORK()` SYSTEM CALL

class sage.parallel.use_fork.**WorkerData**(*input_value*, *starttime=None*, *failure=''*)

Bases: `object`

Simple class which stores data about a running `p_iter_fork` worker.

This just stores three attributes:

- `input` – the input value used by this worker
- `starttime` – the walltime when this worker started
- `failure` – an optional message indicating the kind of failure

EXAMPLES:

```
sage: from sage.parallel.use_fork import WorkerData
sage: W = WorkerData(42); W
<sage.parallel.use_fork.WorkerData object at ...>
sage: W.starttime  # random
1499330252.463206
```

```
>>> from sage.all import *
>>> from sage.parallel.use_fork import WorkerData
>>> W = WorkerData(Integer(42)); W
<sage.parallel.use_fork.WorkerData object at ...>
>>> W.starttime  # random
1499330252.463206
```

class sage.parallel.use_fork.**p_iter_fork**(*ncpus*, *timeout=0*, *verbose=False*, *reset_interfaces=True*, *reseed_rng=False*)

Bases: `object`

A parallel iterator implemented using `fork()`.

INPUT:

- `ncpus` – the maximal number of simultaneous subprocesses to spawn
- `timeout` – float (default: 0); wall time in seconds until a subprocess is automatically killed
- `verbose` – boolean (default: `False`); whether to print anything about what the iterator does (e.g., killing subprocesses)
- `reset_interfaces` – boolean (default: `True`); whether to reset all pexpect interfaces
- `reseed_rng` – boolean (default: `False`); whether or not to reseed the rng in the subprocesses

EXAMPLES:

```
sage: X = sage.parallel.use_fork.p_iter_fork(2,3, False); X
<sage.parallel.use_fork.p_iter_fork object at ...>
sage: X.ncpus
2
sage: X.timeout
3.0
sage: X.verbose
False
```

```
>>> from sage.all import *
>>> X = sage.parallel.use_fork.p_iter_fork(Integer(2),Integer(3), False); X
<sage.parallel.use_fork.p_iter_fork object at ...>
>>> X.ncpus
2
>>> X.timeout
3.0
>>> X.verbose
False
```

CHAPTER

FOUR

PARALLEL COMPUTATIONS USING RECURSIVELYENUMERATEDSET AND MAP-REDUCE

There is an efficient way to distribute computations on a set S of objects defined by `RecursivelyEnumeratedSet()` (see `sage.sets.recursively_enumerated_set` for more details) over which one would like to perform the following kind of operations:

- Compute the cardinality of a (very large) set defined recursively (through a call to `RecursivelyEnumeratedSet_forest`)
- More generally, compute any kind of generating series over this set
- Test a conjecture, e.g. find an element of S satisfying a specific property, or check that none does or that they all do
- Count/list the elements of S that have a specific property
- Apply any map/reduce kind of operation over the elements of S

AUTHORS:

- Florent Hivert – code, documentation (2012–2016)
- Jean Baptiste Priez – prototype, debugging help on MacOSX (2011-June, 2016)
- Nathann Cohen – some documentation (2012)

4.1 Contents

4.2 How is this different from usual MapReduce?

This implementation is specific to `RecursivelyEnumeratedSet_forest`, and uses its properties to do its job. Not only mapping and reducing but also **generating the elements** of S is done on different processors.

4.3 How can I use all that stuff?

First, you need to set the environment variable `SAGE_NUM_THREADS` to the desired number of parallel threads to be used:

```
sage: import os                                    # not tested
sage: os.environ["SAGE_NUM_THREADS"] = '8'         # not tested
```

```
>>> from sage.all import *
>>> import os                                      # not tested
>>> os.environ["SAGE_NUM_THREADS"] = '8'          # not tested
```

Second, you need the information necessary to describe a `RecursivelyEnumeratedSet_forest` representing your set S (see `sage.sets.recursively_enumerated_set`). Then, you need to provide a "map" function as well as a "reduce" function. Here are some examples:

- **Counting the number of elements.** In this situation, the map function can be set to `lambda x: 1`, and the reduce function just adds the values together, i.e. `lambda x, y: x + y`.

  We count binary words of length ≤ 16:

```
sage: seeds = [[]]
sage: succ = lambda l: [l + [0], l + [1]] if len(l) < 16 else []
sage: S = RecursivelyEnumeratedSet(seeds, succ,
....:     structure='forest', enumeration='depth')
sage: map_function = lambda x: 1
sage: reduce_function = lambda x, y: x + y
sage: reduce_init = 0
sage: S.map_reduce(map_function, reduce_function, reduce_init)
131071
```

```
>>> from sage.all import *
>>> seeds = [[]]
>>> succ = lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) < Integer(16)␣
↪else []
>>> S = RecursivelyEnumeratedSet(seeds, succ,
...     structure='forest', enumeration='depth')
>>> map_function = lambda x: Integer(1)
>>> reduce_function = lambda x, y: x + y
>>> reduce_init = Integer(0)
>>> S.map_reduce(map_function, reduce_function, reduce_init)
131071
```

  This matches the number of binary words of length ≤ 16:

```
sage: factor(131071 + 1)
2^17
```

```
>>> from sage.all import *
>>> factor(Integer(131071) + Integer(1))
2^17
```

  Note that the map and reduce functions here have the default values of the `sage.sets.recursively_enumerated_set.RecursivelyEnumeratedSet_forest.map_reduce()` method so that the number of elements can be obtained more simply with:

```
sage: S.map_reduce()
131071
```

```
>>> from sage.all import *
>>> S.map_reduce()
131071
```

Instead of using `RecursivelyEnumeratedSet()`, one can directly use *RESetMapReduce*, which gives finer control over the parallel execution (see *Advanced use* below):

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
....:     roots=[[]],
....:     children=lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
....:     map_function=lambda x: 1,
....:     reduce_function=lambda x, y: x + y,
....:     reduce_init=0)
sage: S.run()
131071
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce(
...     roots=[[]],
...     children=lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) <␣
↪Integer(16) else [],
...     map_function=lambda x: Integer(1),
...     reduce_function=lambda x, y: x + y,
...     reduce_init=Integer(0))
>>> S.run()
131071
```

- **Generating series.** For this, take a Map function that associates a monomial to each element of S, while the Reduce function is still equal to `lambda x, y: x + y`.

  We compute the generating series for counting binary words of each length ≤ 16:

```
sage: S = RecursivelyEnumeratedSet(
....:     [[]], lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
....:     structure='forest', enumeration='depth')
sage: x = polygen(ZZ)
sage: sp = S.map_reduce(
....:     map_function=lambda z: x**len(z),
....:     reduce_function=lambda x, y: x + y,
....:     reduce_init=0)
sage: sp
65536*x^16 + 32768*x^15 + 16384*x^14 + 8192*x^13 + 4096*x^12
+ 2048*x^11 + 1024*x^10 + 512*x^9 + 256*x^8 + 128*x^7 + 64*x^6
+ 32*x^5 + 16*x^4 + 8*x^3 + 4*x^2 + 2*x + 1
```

```
>>> from sage.all import *
>>> S = RecursivelyEnumeratedSet(
...     [[]], lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) <␣
```

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```
↪Integer(16) else [],
...       structure='forest', enumeration='depth')
>>> x = polygen(ZZ)
>>> sp = S.map_reduce(
...       map_function=lambda z: x**len(z),
...       reduce_function=lambda x, y: x + y,
...       reduce_init=Integer(0))
>>> sp
65536*x^16 + 32768*x^15 + 16384*x^14 + 8192*x^13 + 4096*x^12
+ 2048*x^11 + 1024*x^10 + 512*x^9 + 256*x^8 + 128*x^7 + 64*x^6
+ 32*x^5 + 16*x^4 + 8*x^3 + 4*x^2 + 2*x + 1
```

This is of course $\sum_{i=0}^{16}(2x)^i$:

```
sage: sp == sum((2*x)^i for i in range(17))
True
```

```
>>> from sage.all import *
>>> sp == sum((Integer(2)*x)**i for i in range(Integer(17)))
True
```

Here is another example where we count permutations of size ≤ 8 (here we use the default values):

```
sage: S = RecursivelyEnumeratedSet(
....:     [[]],
....:     lambda l: ([l[:i] + [len(l)] + l[i:]
....:                 for i in range(len(l) + 1)] if len(l) < 8 else []),
....:     structure='forest',
....:     enumeration='depth')
sage: x = polygen(ZZ)
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> S = RecursivelyEnumeratedSet(
...       [[]],
...       lambda l: ([l[:i] + [len(l)] + l[i:]
...                   for i in range(len(l) + Integer(1))] if len(l) < Integer(8)␣
↪else []),
...       structure='forest',
...       enumeration='depth')
>>> x = polygen(ZZ)
>>> sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

This is of course $\sum_{i=0}^{8} i!x^i$:

```
sage: sp == sum(factorial(i)*x^i for i in range(9))
True
```

```
>>> from sage.all import *
>>> sp == sum(factorial(i)*x**i for i in range(Integer(9)))
```

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```
True
```

- **Post Processing.** We now demonstrate the use of `post_process`. We generate the permutation as previously, but we only perform the map/reduce computation on those of even `len`. Of course we get the even part of the previous generating series:

```
sage: S = RecursivelyEnumeratedSet(
....:     [[]],
....:     lambda l: ([l[:i] + [len(l) + 1] + l[i:]
....:                 for i in range(len(l) + 1)] if len(l) < 8 else []),
....:     post_process=lambda l: l if len(l) % 2 == 0 else None,
....:     structure='forest',
....:     enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 720*x^6 + 24*x^4 + 2*x^2 + 1
```

```
>>> from sage.all import *
>>> S = RecursivelyEnumeratedSet(
...     [[]],
...     lambda l: ([l[:i] + [len(l) + Integer(1)] + l[i:]
...                 for i in range(len(l) + Integer(1))] if len(l) < Integer(8)␣
↪else []),
...     post_process=lambda l: l if len(l) % Integer(2) == Integer(0) else None,
...     structure='forest',
...     enumeration='depth')
>>> sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 720*x^6 + 24*x^4 + 2*x^2 + 1
```

This is also useful for example to call a constructor on the generated elements:

```
sage: S = RecursivelyEnumeratedSet(
....:     [[]],
....:     lambda l: ([l[:i] + [len(l) + 1] + l[i:]
....:     for i in range(len(l) + 1)] if len(l) < 5 else []),
....:     post_process=lambda l: Permutation(l) if len(l) == 5 else None,
....:     structure='forest',
....:     enumeration='depth')
sage: x = polygen(ZZ)
sage: sp = S.map_reduce(lambda z: x**z.number_of_inversions()); sp
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x +␣
↪1
```

```
>>> from sage.all import *
>>> S = RecursivelyEnumeratedSet(
...     [[]],
...     lambda l: ([l[:i] + [len(l) + Integer(1)] + l[i:]
...     for i in range(len(l) + Integer(1))] if len(l) < Integer(5) else []),
...     post_process=lambda l: Permutation(l) if len(l) == Integer(5) else None,
...     structure='forest',
...     enumeration='depth')
>>> x = polygen(ZZ)
>>> sp = S.map_reduce(lambda z: x**z.number_of_inversions()); sp
```

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```
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x +␣
↪1
```

We get here a polynomial which is the q-factorial (in the variable x) of 5, that is, $\prod_{i=1}^{5} \frac{1-x^i}{1-x}$:

```
sage: x = polygen(ZZ)
sage: prod((1-x^i)//(1-x) for i in range(1, 6))
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x +␣
↪1
```

```
>>> from sage.all import *
>>> x = polygen(ZZ)
>>> prod((Integer(1)-x**i)//(Integer(1)-x) for i in range(Integer(1), Integer(6)))
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x +␣
↪1
```

Compare:

```
sage: from sage.combinat.q_analogues import q_factorial
↪# needs sage.combinat
sage: q_factorial(5)
↪# needs sage.combinat
q^10 + 4*q^9 + 9*q^8 + 15*q^7 + 20*q^6 + 22*q^5 + 20*q^4 + 15*q^3 + 9*q^2 + 4*q +␣
↪1
```

```
>>> from sage.all import *
>>> from sage.combinat.q_analogues import q_factorial                    #␣
↪needs sage.combinat
>>> q_factorial(Integer(5))                                               ␣
↪       # needs sage.combinat
q^10 + 4*q^9 + 9*q^8 + 15*q^7 + 20*q^6 + 22*q^5 + 20*q^4 + 15*q^3 + 9*q^2 + 4*q +␣
↪1
```

- **Listing the objects.** One can also compute the list of objects in a `RecursivelyEnumeratedSet_forest>` using *RESetMapReduce*. As an example, we compute the set of numbers between 1 and 63, generated by their binary expansion:

```
sage: S = RecursivelyEnumeratedSet(
....:     [1],
....:     lambda l: [(l<<1)|0, (l<<1)|1] if l < 1<<5 else [],
....:     structure='forest',
....:     enumeration='depth')
```

```
>>> from sage.all import *
>>> S = RecursivelyEnumeratedSet(
...     [Integer(1)],
...     lambda l: [(l<<Integer(1))|Integer(0), (l<<Integer(1))|Integer(1)] if l <␣
↪Integer(1)<<Integer(5) else [],
...     structure='forest',
...     enumeration='depth')
```

Here is the list computed without *RESetMapReduce*:

```
sage: serial = list(S)
sage: serial
[1, 2, 4, 8, 16, 32, 33, 17, 34, 35, 9, 18, 36, 37, 19, 38, 39, 5, 10,
 20, 40, 41, 21, 42, 43, 11, 22, 44, 45, 23, 46, 47, 3, 6, 12, 24, 48,
 49, 25, 50, 51, 13, 26, 52, 53, 27, 54, 55, 7, 14, 28, 56, 57, 29, 58,
 59, 15, 30, 60, 61, 31, 62, 63]
```

```
>>> from sage.all import *
>>> serial = list(S)
>>> serial
[1, 2, 4, 8, 16, 32, 33, 17, 34, 35, 9, 18, 36, 37, 19, 38, 39, 5, 10,
 20, 40, 41, 21, 42, 43, 11, 22, 44, 45, 23, 46, 47, 3, 6, 12, 24, 48,
 49, 25, 50, 51, 13, 26, 52, 53, 27, 54, 55, 7, 14, 28, 56, 57, 29, 58,
 59, 15, 30, 60, 61, 31, 62, 63]
```

Here is how to perform the parallel computation. The order of the lists depends on the synchronisation of the various computation processes and therefore should be considered as random:

```
sage: parall = S.map_reduce(lambda x: [x], lambda x, y: x + y, [])
sage: parall  # random
[1, 3, 7, 15, 31, 63, 62, 30, 61, 60, 14, 29, 59, 58, 28, 57, 56, 6, 13,
 27, 55, 54, 26, 53, 52, 12, 25, 51, 50, 24, 49, 48, 2, 5, 11, 23, 47,
 46, 22, 45, 44, 10, 21, 43, 42, 20, 41, 40, 4, 9, 19, 39, 38, 18, 37,
 36, 8, 17, 35, 34, 16, 33, 32]
sage: sorted(serial) == sorted(parall)
True
```

```
>>> from sage.all import *
>>> parall = S.map_reduce(lambda x: [x], lambda x, y: x + y, [])
>>> parall  # random
[1, 3, 7, 15, 31, 63, 62, 30, 61, 60, 14, 29, 59, 58, 28, 57, 56, 6, 13,
 27, 55, 54, 26, 53, 52, 12, 25, 51, 50, 24, 49, 48, 2, 5, 11, 23, 47,
 46, 22, 45, 44, 10, 21, 43, 42, 20, 41, 40, 4, 9, 19, 39, 38, 18, 37,
 36, 8, 17, 35, 34, 16, 33, 32]
>>> sorted(serial) == sorted(parall)
True
```

4.4 Advanced use

Fine control over the execution of a map/reduce computation is achieved via parameters passed to the *RESetMapReduce.run()* method. The following three parameters can be used:

- `max_proc` – (integer, default: `None`) if given, the maximum number of worker processors to use. The actual number is also bounded by the value of the environment variable `SAGE_NUM_THREADS` (the number of cores by default).
- `timeout` – a timeout on the computation (default: `None`)
- `reduce_locally` – whether the workers should reduce locally their work or sends results to the master as soon as possible. See *RESetMapReduceWorker* for details.

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import (RESetMPExample, AbortError)
sage: EX = RESetMPExample(maxl=100)
sage: try:
....:     res = EX.run(timeout=float(0.01))
....: except AbortError:
....:     print("Computation timeout")
....: else:
....:     print("Computation normally finished")
....:     res
Computation timeout
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import (RESetMPExample, AbortError)
>>> EX = RESetMPExample(maxl=Integer(100))
>>> try:
...     res = EX.run(timeout=float(RealNumber('0.01')))
... except AbortError:
...     print("Computation timeout")
... else:
...     print("Computation normally finished")
...     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: EX = RESetMPExample(maxl=8)
sage: try:
....:     res = EX.run(timeout=60)
....: except AbortError:
....:     print("Computation Timeout")
....: else:
....:     print("Computation normally finished")
....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> EX = RESetMPExample(maxl=Integer(8))
>>> try:
...     res = EX.run(timeout=Integer(60))
... except AbortError:
...     print("Computation Timeout")
... else:
...     print("Computation normally finished")
...     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

As for `reduce_locally`, one should not see any difference, except for speed during normal usage. Most of the time one should leave it set to `True`, unless one sets up a mechanism to consume the partial results as soon as they arrive. See *`RESetParallelIterator`* and in particular the `__iter__` method for a example of consumer use.

4.5 Profiling

It is possible to profile a map/reduce computation. First we create a *RESetMapReduce* object:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
....:     roots=[[]],
....:     children=lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
....:     map_function=lambda x: 1,
....:     reduce_function=lambda x, y: x + y,
....:     reduce_init=0)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce(
...     roots=[[]],
...     children=lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) <␣
↪Integer(16) else [],
...     map_function=lambda x: Integer(1),
...     reduce_function=lambda x, y: x + y,
...     reduce_init=Integer(0))
```

The profiling is activated by the `profile` parameter. The value provided should be a prefix (including a possible directory) for the profile dump:

```
sage: import tempfile
sage: d = tempfile.TemporaryDirectory(prefix='RESetMR_profile')
sage: res = S.run(profile=d.name)  # random
[RESetMapReduceWorker-1:58] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp1
...
[RESetMapReduceWorker-1:57] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp0
...
sage: res
131071
```

```
>>> from sage.all import *
>>> import tempfile
>>> d = tempfile.TemporaryDirectory(prefix='RESetMR_profile')
>>> res = S.run(profile=d.name)  # random
[RESetMapReduceWorker-1:58] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp1
...
[RESetMapReduceWorker-1:57] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp0
...
>>> res
131071
```

In this example, the profiles have been dumped in files such as `profcomp0`. One can then load and print them as follows. See `cProfile.Profile` for more details:

```
sage: import cProfile, pstats
sage: st = pstats.Stats(d.name+'0')
sage: st.strip_dirs().sort_stats('cumulative').print_stats()  # random
...
   Ordered by: cumulative time

   ncalls  tottime  percall  cumtime  percall filename:lineno(function)
        1    0.023    0.023    0.432    0.432 map_reduce.py:1211(run_myself)
    11968    0.151    0.000    0.223    0.000 map_reduce.py:1292(walk_branch_locally)
...
<pstats.Stats instance at 0x7fedea40c6c8>
```

```
>>> from sage.all import *
>>> import cProfile, pstats
>>> st = pstats.Stats(d.name+'0')
>>> st.strip_dirs().sort_stats('cumulative').print_stats()  # random
...
   Ordered by: cumulative time

   ncalls  tottime  percall  cumtime  percall filename:lineno(function)
        1    0.023    0.023    0.432    0.432 map_reduce.py:1211(run_myself)
    11968    0.151    0.000    0.223    0.000 map_reduce.py:1292(walk_branch_locally)
...
<pstats.Stats instance at 0x7fedea40c6c8>
```

Like a good neighbor we clean up our temporary directory as soon as possible:

```
sage: d.cleanup()
```

```
>>> from sage.all import *
>>> d.cleanup()
```

See also

The Python Profilers for more detail on profiling in python.

4.6 Logging

The computation progress is logged through a `logging.Logger` in `sage.parallel.map_reduce.logger` together with `logging.StreamHandler` and a `logging.Formatter`. They are currently configured to print warning messages to the console.

See also

Logging facility for Python for more detail on logging and log system configuration.

Note

Calls to logger which involve printing the node are commented out in the code, because the printing (to a string) of the node can be very time consuming depending on the node and it happens before the decision whether the logger should record the string or drop it.

4.7 How does it work ?

The scheduling algorithm we use here is any adaptation of Wikipedia article Work_stealing:

> In a work stealing scheduler, each processor in a computer system has a queue of work items (computational tasks, threads) to perform. [...]. Each work items are initially put on the queue of the processor executing the work item. When a processor runs out of work, it looks at the queues of other processors and "steals" their work items. In effect, work stealing distributes the scheduling work over idle processors, and as long as all processors have work to do, no scheduling overhead occurs.

For communication we use Python's basic `multiprocessing` module. We first describe the different actors and communication tools used by the system. The work is done under the coordination of a **master** object (an instance of *`RESetMapReduce`*) by a bunch of **worker** objects (instances of *`RESetMapReduceWorker`*).

Each running map reduce instance works on a `RecursivelyEnumeratedSet_forest>` called here C and is coordinated by a *`RESetMapReduce`* object called the **master**. The master is in charge of launching the work, gathering the results and cleaning up at the end of the computation. It doesn't perform any computation associated to the generation of the element C nor the computation of the mapped function. It however occasionally perform a reduce, but most reducing is by default done by the workers. Also thanks to the work-stealing algorithm, the master is only involved in detecting the termination of the computation but all the load balancing is done at the level of the workers.

Workers are instances of *`RESetMapReduceWorker`*. They are responsible for doing the actual computations: element generation, mapping and reducing. They are also responsible for the load balancing thanks to work-stealing.

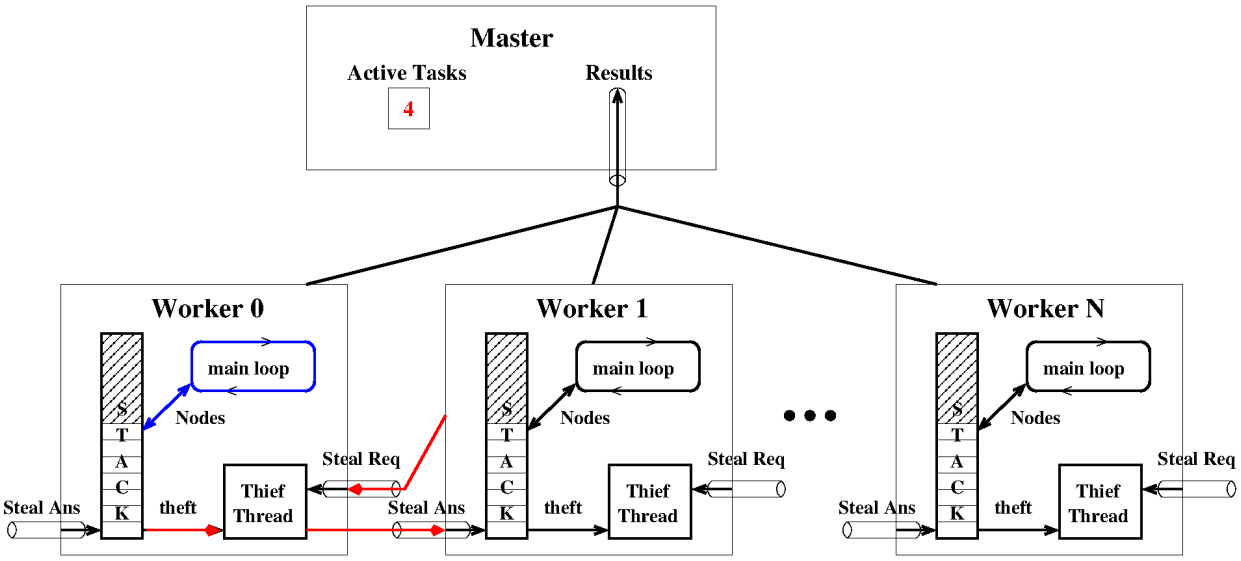
Here is a description of the attributes of the **master** relevant to the map-reduce protocol:

- `_results` – a `SimpleQueue` where the master gathers the results sent by the workers
- `_active_tasks` – a `Semaphore` recording the number of active tasks; the work is complete when it reaches 0
- `_done` – a `Lock` which ensures that shutdown is done only once
- `_aborted` – a `Value()` storing a shared `ctypes.c_bool` which is `True` if the computation was aborted before all workers ran out of work
- `_workers` – list of *`RESetMapReduceWorker`* objects Each worker is identified by its position in this list

Each **worker** is a process (*`RESetMapReduceWorker`* inherits from `Process`) which contains:

- `worker._iproc` – the identifier of the worker that is its position in the master's list of workers
- `worker._todo` – a `collections.deque` storing of nodes of the worker. It is used as a stack by the worker. Thiefs steal from the bottom of this queue.
- `worker._request` – a `SimpleQueue` storing steal request submitted to `worker`
- `worker._read_task`, `worker._write_task` – a `Pipe` used to transfer node during steal
- `worker._thief` – a `Thread` which is in charge of stealing from `worker._todo`

Here is a schematic of the architecture:



4.8 How thefts are performed

During normal time, that is, when all workers are active, a worker `W` is iterating though a loop inside *`RESetMapReduceWorker.walk_branch_locally()`*. Work nodes are taken from and new nodes `W._todo` are appended to `W._todo`. When a worker `W` runs out of work, that is, when `worker._todo` is empty, it tries to steal some work (i.e., a node) from another worker. This is performed in the *`RESetMapReduceWorker.steal()`* method.

From the point of view of `W`, here is what happens:

- `W` signals to the master that it is idle: `master._signal_task_done`;
- `W` chooses a victim `V` at random;
- `W` sends a request to `V`: it puts its identifier into `V._request`;
- `W` tries to read a node from `W._read_task`. Then three things may happen:
  - a proper node is read. Then the theft was a success and `W` starts working locally on the received node.
  - `None` is received. This means that `V` was idle. Then `W` tries another victim.
  - *`AbortError`* is received. This means either that the computation was aborted or that it simply succeeded and that no more work is required by `W`. Therefore an *`AbortError`* exception is raised leading `W` to shutdown.

We now describe the protocol on the victim's side. Each worker process contains a `Thread` which we call `T` for thief which acts like some kind of Troyan horse during theft. It is normally blocked waiting for a steal request.

From the point of view of `V` and `T`, here is what happens:

- during normal time, `T` is blocked waiting on `V._request`;
- upon steal request, `T` wakes up receiving the identification of `W`;
- `T` signals to the master that a new task is starting by `master._signal_task_start`;
- Two things may happen depending if the queue `V._todo` is empty or not. Remark that due to the GIL, there is no parallel execution between the victim `V` and its thief thread `T`.
  - If `V._todo` is empty, then `None` is answered on `W._write_task`. The task is immediately signaled to end the master through `master._signal_task_done`.

- Otherwise, a node is removed from the bottom of `V._todo`. The node is sent to `W` on `W._write_task`. The task will be ended by `W`, that is, when finished working on the subtree rooted at the node, `W` will call `master._signal_task_done`.

4.9 The end of the computation

To detect when a computation is finished, a synchronized integer is kept which counts the number of active tasks. This is essentially a semaphore but semaphores are broken on Darwin OSes so we ship two implementations depending on the OS (see *ActiveTaskCounter* and *ActiveTaskCounterDarwin* and the note below).

When a worker finishes working on a task, it calls `master._signal_task_done`. This decreases the task counter `master._active_tasks`. When it reaches 0, it means that there are no more nodes: the work is completed. The worker executes `master._shutdown` which sends *AbortError* to all `worker._request` and `worker._write_task` queues. Each worker or thief thread receiving such a message raises the corresponding exception, therefore stopping its work. A lock called `master._done` ensures that shutdown is only done once.

Finally, it is also possible to interrupt the computation before its ends, by calling `master.abort()`. This is achieved by setting `master._active_tasks` to 0 and calling `master._shutdown`.

Warning

The macOS Semaphore bug

Darwin OSes do not correctly implement POSIX's semaphore semantic. Indeed, on these systems, acquire may fail and return False not only when the semaphore is equal to zero but also **because someone else is trying to acquire** at the same time. This makes using Semaphores impossible on macOS so that on these systems we use a synchronized integer instead.

4.10 Are there examples of classes?

Yes! Here they are:

- *RESetMPExample* – a simple basic example
- *RESetParallelIterator* – a more advanced example using non standard communication configuration

4.11 Tests

Generating series for the sum of strictly decreasing lists of integers smaller than 15:

```
sage: y = polygen(ZZ, 'y')
sage: R = RESetMapReduce(
....:     roots=[([], 0, 0)] + [([i], i, i) for i in range(1, 15)],
....:     children=lambda list_sum_last:
....:         [(list_sum_last[0] + [i], list_sum_last[1] + i, i)
....:          for i in range(1, list_sum_last[2])],
....:     map_function=lambda li_sum_dummy: y**li_sum_dummy[1])
sage: sg = R.run()
sage: sg == prod((1 + y**i) for i in range(1, 15))
True
```

```
>>> from sage.all import *
>>> y = polygen(ZZ, 'y')
>>> R = RESetMapReduce(
...     roots=[([], Integer(0), Integer(0))] + [([i], i, i) for i in range(Integer(1),
↪ Integer(15))],
...     children=lambda list_sum_last:
...         [(list_sum_last[Integer(0)] + [i], list_sum_last[Integer(1)] + i, i)
...          for i in range(Integer(1), list_sum_last[Integer(2)])],
...     map_function=lambda li_sum_dummy: y**li_sum_dummy[Integer(1)])
>>> sg = R.run()
>>> sg == prod((Integer(1) + y**i) for i in range(Integer(1), Integer(15)))
True
```

4.12 Classes and methods

exception sage.parallel.map_reduce.**AbortError**

Bases: Exception

Exception for aborting parallel computations.

This is used both as exception or as abort message.

sage.parallel.map_reduce.**ActiveTaskCounter**

alias of *ActiveTaskCounterPosix*

class sage.parallel.map_reduce.**ActiveTaskCounterDarwin**(*task_number*)

Bases: object

Handling the number of active tasks.

A class for handling the number of active tasks in a distributed computation process. This is essentially a semaphore, but Darwin OSes do not correctly implement POSIX's semaphore semantic. So we use a shared integer with a lock.

abort()

Set the task counter to zero.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.abort()
sage: c
ActiveTaskCounter(value=0)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
>>> c.abort()
>>> c
ActiveTaskCounter(value=0)
```

task_done()

Decrement the task counter by one.

OUTPUT:

Calling *task_done()* decrements the counter and returns its new value.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_done()
3
sage: c
ActiveTaskCounter(value=3)

sage: c = ATC(0)
sage: c.task_done()
-1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
>>> c.task_done()
3
>>> c
ActiveTaskCounter(value=3)

>>> c = ATC(Integer(0))
>>> c.task_done()
-1
```

task_start()

Increment the task counter by one.

OUTPUT:

Calling *task_start()* on a zero or negative counter returns 0, otherwise increment the counter and returns its value after the incrementation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_start()
5
sage: c
ActiveTaskCounter(value=5)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
```

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```
>>> c.task_start()
5
>>> c
ActiveTaskCounter(value=5)
```

Calling *task_start()* on a zero counter does nothing:

```
sage: c = ATC(0)
sage: c.task_start()
0
sage: c
ActiveTaskCounter(value=0)
```

```
>>> from sage.all import *
>>> c = ATC(Integer(0))
>>> c.task_start()
0
>>> c
ActiveTaskCounter(value=0)
```

class sage.parallel.map_reduce.**ActiveTaskCounterPosix**(*task_number*)

Bases: object

Handling the number of active tasks.

A class for handling the number of active tasks in a distributed computation process. This is the standard implementation on POSIX compliant OSes. We essentially wrap a semaphore.

Note

A legitimate question is whether there is a need in keeping the two implementations. I ran the following experiment on my machine:

```
S = RecursivelyEnumeratedSet(
        [[]],
        lambda l: ([l[:i] + [len(l)] + l[i:]
                    for i in range(len(l) + 1)]
                   if len(l) < NNN else []),
        structure='forest',
        enumeration='depth')
%time sp = S.map_reduce(lambda z: x**len(z)); sp
```

For NNN = 10, averaging a dozen of runs, I got:

- Posix compliant implementation: 17.04 s
- Darwin implementation: 18.26 s

So there is a non negligible overhead. It will probably be worth it if we try to cythonize the code. So I'm keeping both implementations.

abort()

Set the task counter to zero.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounter as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.abort()
sage: c
ActiveTaskCounter(value=0)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounter as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
>>> c.abort()
>>> c
ActiveTaskCounter(value=0)
```

task_done()

Decrement the task counter by one.

OUTPUT:

Calling *task_done()* decrements the counter and returns its new value.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounter as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_done()
3
sage: c
ActiveTaskCounter(value=3)

sage: c = ATC(0)
sage: c.task_done()
-1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounter as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
>>> c.task_done()
3
>>> c
ActiveTaskCounter(value=3)

>>> c = ATC(Integer(0))
>>> c.task_done()
-1
```

task_start()

Increment the task counter by one.

OUTPUT:

Calling *task_start()* on a zero or negative counter returns 0, otherwise increment the counter and returns its value after the incrementation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_start()
5
sage: c
ActiveTaskCounter(value=5)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
>>> c = ATC(Integer(4)); c
ActiveTaskCounter(value=4)
>>> c.task_start()
5
>>> c
ActiveTaskCounter(value=5)
```

Calling *task_start()* on a zero counter does nothing:

```
sage: c = ATC(0)
sage: c.task_start()
0
sage: c
ActiveTaskCounter(value=0)
```

```
>>> from sage.all import *
>>> c = ATC(Integer(0))
>>> c.task_start()
0
>>> c
ActiveTaskCounter(value=0)
```

class sage.parallel.map_reduce.**RESetMPExample**(*maxl=9*)

Bases: *RESetMapReduce*

An example of map reduce class.

INPUT:

- `maxl` – the maximum size of permutations generated (default: 9)

This computes the generating series of permutations counted by their size up to size `maxl`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample()
sage: EX.run()
362880*x^9 + 40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5
+ 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> EX = RESetMPExample()
>>> EX.run()
362880*x^9 + 40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5
+ 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

See also

This is an example of *RESetMapReduce*

children(*l*)

Return the children of the permutation *l*.

INPUT:

- `l` – list containing a permutation

OUTPUT:

The lists with `len(l)` inserted at all possible positions into `l`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().children([1,0])
[[2, 1, 0], [1, 2, 0], [1, 0, 2]]
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> RESetMPExample().children([Integer(1),Integer(0)])
[[2, 1, 0], [1, 2, 0], [1, 0, 2]]
```

map_function(*l*)

The monomial associated to the permutation *l*.

INPUT:

- `l` – list containing a permutation

OUTPUT:

The monomial `x^len(l)`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().map_function([1,0])
x^2
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> RESetMPExample().map_function([Integer(1),Integer(0)])
x^2
```

roots()

Return the empty permutation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().roots()
[[]]
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> RESetMPExample().roots()
[[]]
```

class sage.parallel.map_reduce.**RESetMapReduce**(*roots=None*, *children=None*, *post_process=None*, *map_function=None*, *reduce_function=None*, *reduce_init=None*, *forest=None*)

Bases: object

Map-Reduce on recursively enumerated sets.

INPUT:

Description of the set:

- either forest=f – where f is a RecursivelyEnumeratedSet_forest>
- or a triple roots, children, post_process as follows
  - roots=r – the root of the enumeration
  - children=c – a function iterating through children nodes, given a parent node
  - post_process=p – a post-processing function

The option post_process allows for customizing the nodes that are actually produced. Furthermore, if post_process(x) returns None, then x won't be output at all.

Description of the map/reduce operation:

- map_function=f – (default: None)
- reduce_function=red – (default: None)
- reduce_init=init – (default: None)

See also

the Map/Reduce module for details and examples.

abort()

Abort the current parallel computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator([[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 17 else [])
sage: it = iter(S)
```

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```
sage: next(it)  # random
[]
sage: S.abort()
sage: hasattr(S, 'work_queue')
False
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetParallelIterator
>>> S = RESetParallelIterator([[]],
...     lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) <␣
↪Integer(17) else [])
>>> it = iter(S)
>>> next(it)  # random
[]
>>> S.abort()
>>> hasattr(S, 'work_queue')
False
```

Cleanup:

```
sage: S.finish()
```

```
>>> from sage.all import *
>>> S.finish()
```

finish()

Destroy the workers and all the communication objects.

Communication statistics are gathered before destroying the workers.

See also

print_communication_statistics()

get_results(*timeout=None*)

Get the results from the queue.

OUTPUT:

The reduction of the results of all the workers, that is, the result of the map/reduce computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: for v in [1, 2, None, 3, None]: S._results.put(v)
sage: S.get_results()
6
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
```

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```
>>> S = RESetMapReduce()
>>> S.setup_workers(Integer(2))
>>> for v in [Integer(1), Integer(2), None, Integer(3), None]: S._results.
↪put(v)
>>> S.get_results()
6
```

Cleanup:

```
sage: del S._results, S._active_tasks, S._done, S._workers
```

```
>>> from sage.all import *
>>> del S._results, S._active_tasks, S._done, S._workers
```

map_function(*o*)

Return the function mapped by `self`.

INPUT:

- `o` – a node

OUTPUT: by default `1`

Note

This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.map_function(7)
1
sage: S = RESetMapReduce(map_function = lambda x: 3*x + 5)
sage: S.map_function(7)
26
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce()
>>> S.map_function(Integer(7))
1
>>> S = RESetMapReduce(map_function = lambda x: Integer(3)*x + Integer(5))
>>> S.map_function(Integer(7))
26
```

post_process(*a*)

Return the image of `a` under the post-processing function for `self`.

INPUT:

- `a` – a node

With the default post-processing function, which is the identity function, this returns `a` itself.

Note

This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.post_process(4)
4
sage: S = RESetMapReduce(post_process=lambda x: x*x)
sage: S.post_process(4)
16
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce()
>>> S.post_process(Integer(4))
4
>>> S = RESetMapReduce(post_process=lambda x: x*x)
>>> S.post_process(Integer(4))
16
```

print_communication_statistics(*blocksize=16*)

Print the communication statistics in a nice way.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=6)
sage: S.run()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

sage: S.print_communication_statistics()  # random
#proc:         0    1    2    3    4    5    6    7
reqs sent:     5    2    3   11   21   19    1    0
reqs rcvs:    10   10    9    5    1   11    9    2
- thefs:       1    0    0    0    0    0    0    0
+ thefs:       0    0    1    0    0    0    0    0
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> S = RESetMPExample(maxl=Integer(6))
>>> S.run()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

>>> S.print_communication_statistics()  # random
#proc:         0    1    2    3    4    5    6    7
reqs sent:     5    2    3   11   21   19    1    0
reqs rcvs:    10   10    9    5    1   11    9    2
- thefs:       1    0    0    0    0    0    0    0
+ thefs:       0    0    1    0    0    0    0    0
```

random_worker()

Return a random worker.

OUTPUT: a worker for `self` chosen at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(2)
sage: EX.random_worker()
<RESetMapReduceWorker...RESetMapReduceWorker-... initial...>
sage: EX.random_worker() in EX._workers
True
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> from threading import Thread
>>> EX = RESetMPExample(maxl=Integer(6))
>>> EX.setup_workers(Integer(2))
>>> EX.random_worker()
<RESetMapReduceWorker...RESetMapReduceWorker-... initial...>
>>> EX.random_worker() in EX._workers
True
```

Cleanup:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

```
>>> from sage.all import *
>>> del EX._results, EX._active_tasks, EX._done, EX._workers
```

reduce_function(*a*, *b*)

Return the reducer function for `self`.

INPUT:

- `a`, `b` – two values to be reduced

OUTPUT: by default the sum of `a` and `b`

Note

This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.reduce_function(4, 3)
7
sage: S = RESetMapReduce(reduce_function=lambda x,y: x*y)
```

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```
sage: S.reduce_function(4, 3)
12
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce()
>>> S.reduce_function(Integer(4), Integer(3))
7
>>> S = RESetMapReduce(reduce_function=lambda x,y: x*y)
>>> S.reduce_function(Integer(4), Integer(3))
12
```

reduce_init()

Return the initial element for a reduction.

Note

This should be overloaded in applications.

roots()

Return the roots of `self`.

OUTPUT: an iterable of nodes

Note

This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(42)
sage: S.roots()
42
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMapReduce
>>> S = RESetMapReduce(Integer(42))
>>> S.roots()
42
```

run(*max_proc=None*, *reduce_locally=True*, *timeout=None*, *profile=None*)

Run the computations.

INPUT:

- `max_proc` – (integer, default: `None`) if given, the maximum number of worker processors to use. The actual number is also bounded by the value of the environment variable `SAGE_NUM_THREADS` (the number of cores by default).
- `reduce_locally` – see *RESetMapReduceWorker* (default: `True`)

- `timeout` – a timeout on the computation (default: `None`)
- `profile` – directory/filename prefix for profiling, or `None` for no profiling (default: `None`)

OUTPUT:

The result of the map/reduce computation or an exception *AbortError* if the computation was interrupted or timeout.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 8)
sage: EX.run()
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> EX = RESetMPExample(maxl = Integer(8))
>>> EX.run()
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import AbortError
sage: EX = RESetMPExample(maxl = 100)
sage: try:
....:     res = EX.run(timeout=float(0.01))
....: except AbortError:
....:     print("Computation timeout")
....: else:
....:     print("Computation normally finished")
....:     res
Computation timeout
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import AbortError
>>> EX = RESetMPExample(maxl = Integer(100))
>>> try:
...     res = EX.run(timeout=float(RealNumber('0.01')))
... except AbortError:
...     print("Computation timeout")
... else:
...     print("Computation normally finished")
...     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: from sage.parallel.map_reduce import AbortError
sage: EX = RESetMPExample(maxl = 8)
sage: try:
....:     res = EX.run(timeout=60)
....: except AbortError:
....:     print("Computation Timeout")
```

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```
....: else:
....:     print("Computation normally finished")
....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import AbortError
>>> EX = RESetMPExample(maxl = Integer(8))
>>> try:
...     res = EX.run(timeout=Integer(60))
... except AbortError:
...     print("Computation Timeout")
... else:
...     print("Computation normally finished")
...     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

run_serial()

Run the computation serially (mostly for tests).

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 4)
sage: EX.run_serial()
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample
>>> EX = RESetMPExample(maxl = Integer(4))
>>> EX.run_serial()
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

setup_workers(*max_proc=None*, *reduce_locally=True*)

Setup the communication channels.

INPUT:

- `max_proc` – integer; an upper bound on the number of worker processes
- `reduce_locally` – whether the workers should reduce locally their work or sends results to the master as soon as possible. See *RESetMapReduceWorker* for details.

start_workers()

Launch the workers.

The workers should have been created using *setup_workers()*.

class sage.parallel.map_reduce.**RESetMapReduceWorker**(*mapred*, *iproc*, *reduce_locally*)

Bases: `ForkProcess`

Worker for generate-map-reduce.

This shouldn't be called directly, but instead created by *RESetMapReduce.setup_workers()*.

INPUT:

- `mapred` – the instance of *RESetMapReduce* for which this process is working
- `iproc` – the id of this worker
- `reduce_locally` – when reducing the results. Three possible values are supported:
  - `True` – means the reducing work is done all locally, the result is only sent back at the end of the work. This ensure the lowest level of communication.
  - `False` – results are sent back after each finished branches, when the process is asking for more work.

run()

The main function executed by the worker.

Calls *run_myself()* after possibly setting up parallel profiling.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])

sage: w.run()
sage: sleep(int(1))
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> EX = RESetMPExample(maxl=Integer(6))
>>> EX.setup_workers(Integer(1))

>>> w = EX._workers[Integer(0)]
>>> w._todo.append(EX.roots()[Integer(0)])

>>> w.run()
>>> sleep(int(Integer(1)))
>>> w._todo.append(None)

>>> EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

```
>>> from sage.all import *
>>> del EX._results, EX._active_tasks, EX._done, EX._workers
```

run_myself()

The main function executed by the worker.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])
sage: w.run_myself()

sage: sleep(int(1))
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> EX = RESetMPExample(maxl=Integer(6))
>>> EX.setup_workers(Integer(1))

>>> w = EX._workers[Integer(0)]
>>> w._todo.append(EX.roots()[Integer(0)])
>>> w.run_myself()

>>> sleep(int(Integer(1)))
>>> w._todo.append(None)

>>> EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

```
>>> from sage.all import *
>>> del EX._results, EX._active_tasks, EX._done, EX._workers
```

send_partial_result()

Send results to the MapReduce process.

Send the result stored in `self._res` to the master and reinitialize it to `master.reduce_init`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: EX.setup_workers(1)
sage: w = EX._workers[0]
```

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```
sage: w._res = 4
sage: w.send_partial_result()
sage: w._res
0
sage: EX._results.get()
4
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> EX = RESetMPExample(maxl=Integer(4))
>>> EX.setup_workers(Integer(1))
>>> w = EX._workers[Integer(0)]
>>> w._res = Integer(4)
>>> w.send_partial_result()
>>> w._res
0
>>> EX._results.get()
4
```

steal()

Steal some node from another worker.

OUTPUT: a node stolen from another worker chosen at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(2)

sage: # known bug (Issue #27537)
sage: w0, w1 = EX._workers
sage: w0._todo.append(42)
sage: thief0 = Thread(target = w0._thief, name='Thief')
sage: thief0.start()
sage: w1.steal()
42
sage: w0._todo
deque([])
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> from threading import Thread
>>> EX = RESetMPExample(maxl=Integer(6))
>>> EX.setup_workers(Integer(2))

>>> # known bug (Issue #27537)
>>> w0, w1 = EX._workers
>>> w0._todo.append(Integer(42))
>>> thief0 = Thread(target = w0._thief, name='Thief')
```

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```
>>> thief0.start()
>>> w1.steal()
42
>>> w0._todo
deque([])
```

walk_branch_locally(*node*)

Work locally.

Performs the map/reduce computation on the subtrees rooted at `node`.

INPUT:

- `node` – the root of the subtree explored

OUTPUT: nothing, the result are stored in `self._res`

This is where the actual work is performed.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample,␣
↪RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: w = RESetMapReduceWorker(EX, 0, True)
sage: def sync(): pass
sage: w.synchronize = sync
sage: w._res = 0

sage: w.walk_branch_locally([])
sage: w._res
x^4 + x^3 + x^2 + x + 1

sage: w.walk_branch_locally(w._todo.pop())
sage: w._res
2*x^4 + x^3 + x^2 + x + 1

sage: while True: w.walk_branch_locally(w._todo.pop())
Traceback (most recent call last):
...
IndexError: pop from an empty deque
sage: w._res
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
>>> EX = RESetMPExample(maxl=Integer(4))
>>> w = RESetMapReduceWorker(EX, Integer(0), True)
>>> def sync(): pass
>>> w.synchronize = sync
>>> w._res = Integer(0)

>>> w.walk_branch_locally([])
>>> w._res
x^4 + x^3 + x^2 + x + 1
```

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```
>>> w.walk_branch_locally(w._todo.pop())
>>> w._res
2*x^4 + x^3 + x^2 + x + 1

>>> while True: w.walk_branch_locally(w._todo.pop())
Traceback (most recent call last):
...
IndexError: pop from an empty deque
>>> w._res
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

class sage.parallel.map_reduce.**RESetParallelIterator**(*roots=None*, *children=None*, *post_process=None*, *map_function=None*, *reduce_function=None*, *reduce_init=None*, *forest=None*)

Bases: *RESetMapReduce*

A parallel iterator for recursively enumerated sets.

This demonstrates how to use *RESetMapReduce* to get an iterator on a recursively enumerated set for which the computations are done in parallel.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator([[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 15 else [])
sage: sum(1 for _ in S)
65535
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetParallelIterator
>>> S = RESetParallelIterator([[]],
...     lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) < Integer(15)␣
↪else [])
>>> sum(Integer(1) for _ in S)
65535
```

map_function(*z*)

Return a singleton tuple.

INPUT:

- `z` – a node

OUTPUT:

The singleton `(z, )`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 15 else [])
```

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```
sage: S.map_function([1, 0])
([1, 0],)
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import RESetParallelIterator
>>> S = RESetParallelIterator( [[]],
...     lambda l: [l + [Integer(0)], l + [Integer(1)]] if len(l) <␣
↪Integer(15) else [])
>>> S.map_function([Integer(1), Integer(0)])
([1, 0],)
```

reduce_init

alias of `tuple`

`sage.parallel.map_reduce.`**proc_number**(*max_proc=None*)

Return the number of processes to use.

INPUT:

- `max_proc` – an upper bound on the number of processes or `None`

EXAMPLES:

```
sage: from sage.parallel.map_reduce import proc_number
sage: proc_number()  # random
8
sage: proc_number(max_proc=1)
1
sage: proc_number(max_proc=2) in (1, 2)
True
```

```
>>> from sage.all import *
>>> from sage.parallel.map_reduce import proc_number
>>> proc_number()  # random
8
>>> proc_number(max_proc=Integer(1))
1
>>> proc_number(max_proc=Integer(2)) in (Integer(1), Integer(2))
True
```

CHAPTER

FIVE

PARALLEL ITERATOR BUILT USING PYTHON'S MULTIPROCESSING MODULE

sage.parallel.multiprocessing_sage.**parallel_iter**(*processes*, *f*, *inputs*)

Return a parallel iterator.

INPUT:

- `processes` – integer
- `f` – function
- `inputs` – an iterable of pairs (args, kwds)

OUTPUT: iterator over values of `f` at `args, kwds` in some random order

EXAMPLES:

```
sage: def f(x): return x+x
sage: import sage.parallel.multiprocessing_sage
sage: v = list(sage.parallel.multiprocessing_sage.parallel_iter(2, f, [((2,), {}),
↪ ((3,),{})]))
sage: v.sort(); v
[(((2,), {}), 4), (((3,), {}), 6)]
```

```
>>> from sage.all import *
>>> def f(x): return x+x
>>> import sage.parallel.multiprocessing_sage
>>> v = list(sage.parallel.multiprocessing_sage.parallel_iter(Integer(2), f,␣
↪[((Integer(2),), {}), ((Integer(3),),{})]))
>>> v.sort(); v
[(((2,), {}), 4), (((3,), {}), 6)]
```

sage.parallel.multiprocessing_sage.**pyprocessing**(*processes=0*)

Return a parallel iterator using a given number of processes implemented using pyprocessing.

INPUT:

- `processes` – integer (default: 0); if 0, set to the number of processors on the computer

OUTPUT: a (partially evaluated) function

EXAMPLES:

```
sage: from sage.parallel.multiprocessing_sage import pyprocessing
sage: p_iter = pyprocessing(4)
```

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```
sage: P = parallel(p_iter=p_iter)
sage: def f(x): return x+x
sage: v = list(P(f)(list(range(10)))); v.sort(); v
[(((0,), {}), 0), (((1,), {}), 2), (((2,), {}), 4), (((3,), {}), 6), (((4,), {}),␣
↪8), (((5,), {}), 10), (((6,), {}), 12), (((7,), {}), 14), (((8,), {}), 16),␣
↪(((9,), {}), 18)]
```

```
>>> from sage.all import *
>>> from sage.parallel.multiprocessing_sage import pyprocessing
>>> p_iter = pyprocessing(Integer(4))
>>> P = parallel(p_iter=p_iter)
>>> def f(x): return x+x
>>> v = list(P(f)(list(range(Integer(10))))); v.sort(); v
[(((0,), {}), 0), (((1,), {}), 2), (((2,), {}), 4), (((3,), {}), 6), (((4,), {}),␣
↪8), (((5,), {}), 10), (((6,), {}), 12), (((7,), {}), 14), (((8,), {}), 16),␣
↪(((9,), {}), 18)]
```

CHAPTER

SIX

PARALLELIZATION CONTROL

This module defines the singleton class *Parallelism* to govern the parallelization of computations in some specific topics. It allows the user to set the number of processes to be used for parallelization.

Some examples of use are provided in the documentation of `sage.tensor.modules.comp.Components.contract()`.

AUTHORS:

- Marco Mancini, Eric Gourgoulhon, Michal Bejger (2015): initial version

class sage.parallel.parallelism.**Parallelism**

Bases: `Singleton`, `SageObject`

Singleton class for managing the number of processes used in parallel computations involved in various fields.

EXAMPLES:

The number of processes is initialized to 1 (no parallelization) for each field:

```
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

```
>>> from sage.all import *
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

Using 4 processes to parallelize tensor computations:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 4
sage: Parallelism().get('tensor')
4
```

```
>>> from sage.all import *
>>> Parallelism().set('tensor', nproc=Integer(4))
>>> Parallelism()
```

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```
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 4
>>> Parallelism().get('tensor')
4
```

Using 6 processes to parallelize all types of computations:

```
sage: Parallelism().set(nproc=6)
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 6
 - tensor computations: 6
```

```
>>> from sage.all import *
>>> Parallelism().set(nproc=Integer(6))
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 6
 - tensor computations: 6
```

Using all the cores available on the computer to parallelize tensor computations:

```
sage: Parallelism().set('tensor')
sage: Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 8
```

```
>>> from sage.all import *
>>> Parallelism().set('tensor')
>>> Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 8
```

Using all the cores available on the computer to parallelize all types of computations:

```
sage: Parallelism().set()
sage: Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 8
 - tensor computations: 8
```

```
>>> from sage.all import *
>>> Parallelism().set()
>>> Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 8
 - tensor computations: 8
```

Switching off all parallelizations:

```
sage: Parallelism().set(nproc=1)
```

```
>>> from sage.all import *
>>> Parallelism().set(nproc=Integer(1))
```

get(*field*)

Return the number of processes which will be used in parallel computations regarding some specific field.

INPUT:

- `field` – string specifying the part of Sage involved in parallel computations

OUTPUT:

- number of processes used in parallelization of computations pertaining to `field`

EXAMPLES:

The default is a single process (no parallelization):

```
sage: Parallelism().reset()
sage: Parallelism().get('tensor')
1
```

```
>>> from sage.all import *
>>> Parallelism().reset()
>>> Parallelism().get('tensor')
1
```

Asking for parallelization on 4 cores:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism().get('tensor')
4
```

```
>>> from sage.all import *
>>> Parallelism().set('tensor', nproc=Integer(4))
>>> Parallelism().get('tensor')
4
```

get_all()

Return the number of processes which will be used in parallel computations in all fields

OUTPUT:

- dictionary of the number of processes, with the computational fields as keys

EXAMPLES:

```
sage: Parallelism().reset()
sage: Parallelism().get_all()
{'linbox': 1, 'tensor': 1}
```

```
>>> from sage.all import *
>>> Parallelism().reset()
>>> Parallelism().get_all()
{'linbox': 1, 'tensor': 1}
```

Asking for parallelization on 4 cores:

```
sage: Parallelism().set(nproc=4)
sage: Parallelism().get_all()
{'linbox': 4, 'tensor': 4}
```

```
>>> from sage.all import *
>>> Parallelism().set(nproc=Integer(4))
>>> Parallelism().get_all()
{'linbox': 4, 'tensor': 4}
```

get_default()

Return the default number of processes to be launched in parallel computations.

EXAMPLES:

A priori, the default number of process for parallelization is the total number of cores found on the computer:

```
sage: Parallelism().reset()
sage: Parallelism().get_default()  # random (depends on the computer)
8
```

```
>>> from sage.all import *
>>> Parallelism().reset()
>>> Parallelism().get_default()  # random (depends on the computer)
8
```

It can be changed via *set_default()*:

```
sage: Parallelism().set_default(nproc=4)
sage: Parallelism().get_default()
4
```

```
>>> from sage.all import *
>>> Parallelism().set_default(nproc=Integer(4))
>>> Parallelism().get_default()
4
```

reset()

Put the singleton object `Parallelism()` in the same state as immediately after its creation.

EXAMPLES:

State of `Parallelism()` just after its creation:

```
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
sage: Parallelism().get_default()  # random (depends on the computer)
8
```

```
>>> from sage.all import *
>>> Parallelism()
```

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```
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
>>> Parallelism().get_default()  # random (depends on the computer)
8
```

Changing some values:

```
sage: Parallelism().set_default(6)
sage: Parallelism().set()
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 6
 - tensor computations: 6
sage: Parallelism().get_default()
6
```

```
>>> from sage.all import *
>>> Parallelism().set_default(Integer(6))
>>> Parallelism().set()
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 6
 - tensor computations: 6
>>> Parallelism().get_default()
6
```

Back to the initial state:

```
sage: Parallelism().reset()
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
sage: Parallelism().get_default()  # random (depends on the computer)
8
```

```
>>> from sage.all import *
>>> Parallelism().reset()
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
>>> Parallelism().get_default()  # random (depends on the computer)
8
```

set(*field=None*, *nproc=None*)

Set the number of processes to be launched for parallel computations regarding some specific field.

INPUT:

- `field` – (default: `None`) string specifying the computational field for which the number of parallel processes is to be set; if `None`, all fields are considered

- `nproc` – (default: `None`) number of processes to be used for parallelization; if `None`, the number of processes will be set to the default value, which, unless redefined by *`set_default()`*, is the total number of cores found on the computer.

EXAMPLES:

The default is a single processor (no parallelization):

```
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

```
>>> from sage.all import *
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

Asking for parallelization on 4 cores in tensor algebra:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 4
```

```
>>> from sage.all import *
>>> Parallelism().set('tensor', nproc=Integer(4))
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 4
```

Using all the cores available on the computer:

```
sage: Parallelism().set('tensor')
sage: Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 8
```

```
>>> from sage.all import *
>>> Parallelism().set('tensor')
>>> Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 8
```

Using 6 cores in all parallelizations:

```
sage: Parallelism().set(nproc=6)
sage: Parallelism()
Number of processes for parallelization:
```

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```
 - linbox computations: 6
 - tensor computations: 6
```

```
>>> from sage.all import *
>>> Parallelism().set(nproc=Integer(6))
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 6
 - tensor computations: 6
```

Using all the cores available on the computer in all parallelizations:

```
sage: Parallelism().set()
sage: Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 8
 - tensor computations: 8
```

```
>>> from sage.all import *
>>> Parallelism().set()
>>> Parallelism()  # random (depends on the computer)
Number of processes for parallelization:
 - linbox computations: 8
 - tensor computations: 8
```

Switching off the parallelization:

```
sage: Parallelism().set(nproc=1)
sage: Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

```
>>> from sage.all import *
>>> Parallelism().set(nproc=Integer(1))
>>> Parallelism()
Number of processes for parallelization:
 - linbox computations: 1
 - tensor computations: 1
```

set_default(*nproc=None*)

Set the default number of processes to be launched in parallel computations.

INPUT:

- `nproc` – (default: `None`) default number of processes; if `None`, the number of processes will be set to the total number of cores found on the computer.

EXAMPLES:

A priori the default number of process for parallelization is the total number of cores found on the computer:

```
sage: Parallelism().get_default()  # random (depends on the computer)
8
```

```
>>> from sage.all import *
>>> Parallelism().get_default()  # random (depends on the computer)
8
```

Changing it thanks to `set_default`:

```
sage: Parallelism().set_default(nproc=4)
sage: Parallelism().get_default()
4
```

```
>>> from sage.all import *
>>> Parallelism().set_default(nproc=Integer(4))
>>> Parallelism().get_default()
4
```

Setting it back to the total number of cores available on the computer:

```
sage: Parallelism().set_default()
sage: Parallelism().get_default()  # random (depends on the computer)
8
```

```
>>> from sage.all import *
>>> Parallelism().set_default()
>>> Parallelism().get_default()  # random (depends on the computer)
8
```

CHAPTER

SEVEN

CPU DETECTION

sage.parallel.ncpus.**ncpus**()

Return the number of available CPUs in the system.

ALGORITHM: `os.sched_getaffinity()` or `os.cpu_count()`

EXAMPLES:

```
sage: sage.parallel.ncpus.ncpus()  # random output -- depends on machine
2
```

```
>>> from sage.all import *
>>> sage.parallel.ncpus.ncpus()  # random output -- depends on machine
2
```

See also

- `sage.interfaces.psage`

CHAPTER

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